

MILANKOVITCH VERSUS CHAOS: OPPOSING OR COMPLEMENTARY THEORIES?

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Introduction

The analytical phase of quantitative Palaeoecology - Palaeoceanography started in the middle seventies with the development of an hypothetical-deductive model which seeks to interpret and explain rhythmical patterns on high resolution proxy time series as a linear response to a global mechanism known as orbital forcing. Directly related to the theory proposed in the thirties by the Serbian mathematician and astronomer Milutin Milankovitch, according to this mechanism Pleistocene glacial cycles, were produced by variations of the solar radiation flux reaching our planet (Berger *et al.*, 1984). The notion that these variations could be related to cyclic geometric changes in the earth's orbit began with J. Adhemar, in 1842, who suggested that the main cause for the "ice ages" could be variations in the direction of earth's orbital axis and, consequently, in insolation (Berger & Pestiaux, 1984).

Such theories usually assume that climate and the ecological systems in general have a linear response to the regular and periodic oscillatory behaviour of several Earth orbital parameters, *i.e.* the proxy data response from (palaeo)climate or (palaeo)oceanographic dynamics is more or less proportional to the magnitude of the orbitally induced periodic disturbance (Imbrie *et al.*, 1984; Herterich & Sarnthein, 1984). This linear approach to Milankovitch theory will be referred to here as the "Periodic behaviour model" (Cachão, 1992).

The search for numerical solutions for the equations that rule our planetary system goes back to the first attempts of the French mathematician Pierre Simon de Laplace to solve an important issue: the Solar System's stability. By making some simplifications Laplace showed that his simplified system was integrable and that there where long-term (tens of thousands of years) periodicities (Murray, 1992).

The present day values for the periodic variations of the maximum solar energy that reaches our planet correspond to one of the trigonometric solutions of the Lagrangean equations for the planetary movements, which are (Berger, 1984):

- 400 and 100 ka, periodicities associated with sinusoidal variations of the earth orbit eccentricity around the Sun;

- 41 ka, a period related to variations on Earth rotation axis obliquity;

- 23 and 19 ka, both periods related to variations in the precession of the equinoxes.

These values are only valid for an interval of time

that goes back to -5 Ma (earliest Pliocene) due to inherent limitations to the calculations involved (*op. cit.*) and to the existence of multiple disturbance factors such as solid tides, solar wind, and meteoritic showers (Buys & Ghil, 1984). So several corrections have been introduced and new values proposed (*e.g.* Berger *et al.*, 1989) for other time intervals.

Milankovitch frequencies result from the nonlinear interference of the several gravitational forces involved in movement of our planetary system. The importance in recognising these frequencies in our time series data is obvious: it would increase our biostratigraphic resolution to a degree unthinkable a few years ago. But what does it strictly tell us about how the natural systems (climate, oceanic currents - the "conveyor belt" of Broecker & Denton, 1990)-, upwelling and the "El Niño" events, population dynamics, ecological succession) work? Not much. In fact the linear approach to the Milankovitch model assumes that the environment is "transparent" or passive to the orbital frequency signals. But our everyday experience suggests the opposite.

There are no natural dynamic systems that can be illustrated by a straight line, or in other words, natural processes are all nonlinear¹. By nonlinear we mean "with feedback" and the full consequences of this are only now being understood through the application of Chaos Theory, since the first discoveries of Eduard Lorenz, a meteorologist from the Massachusetts Institute of Technology, in the early 1960s (Lorenz, 1963).

The main purpose of this essay is to contribute to the development of an alternative model that may find room to explain, as a gestalt theory, not only data series with rhythmic patterns but also alternating processes without a strict periodicity.

To discuss whether there is "Milankovitch" or not is a false problem. Orbital forcing has always been present as a pumping mechanism on Earth environment. Whether its frequencies are stable or change to some degree, is another problem which will not be addressed here. What we should discuss is under what conditions should we expect to recognise these periodic signals in our data and in which conditions should we expect not to see them, and why?

An alternative approach

The model, which is proposed as an alternative to the linear "Periodic behaviour model", was designated as "Complex behaviour model" (Cachão, 1992). As Broecker & Denton wrote (1990) this "proposal is not a rejection

¹ - Although for some authors the system's response seems to behave "nearly linearly" in the 43 and 23-ka frequencies Imbrie (1992) recently recognised that the 100-ka cycle "might either reflect an oscillation driven by non linear interactions occurring within the system itself [vide Self-periodic sector in Fig. 3 of this essay], or an interaction occurring between the system and the astronomically forced responses" simply because the "100,000-year cycle of radiation is much too small to be effective" (*op. cit.*).

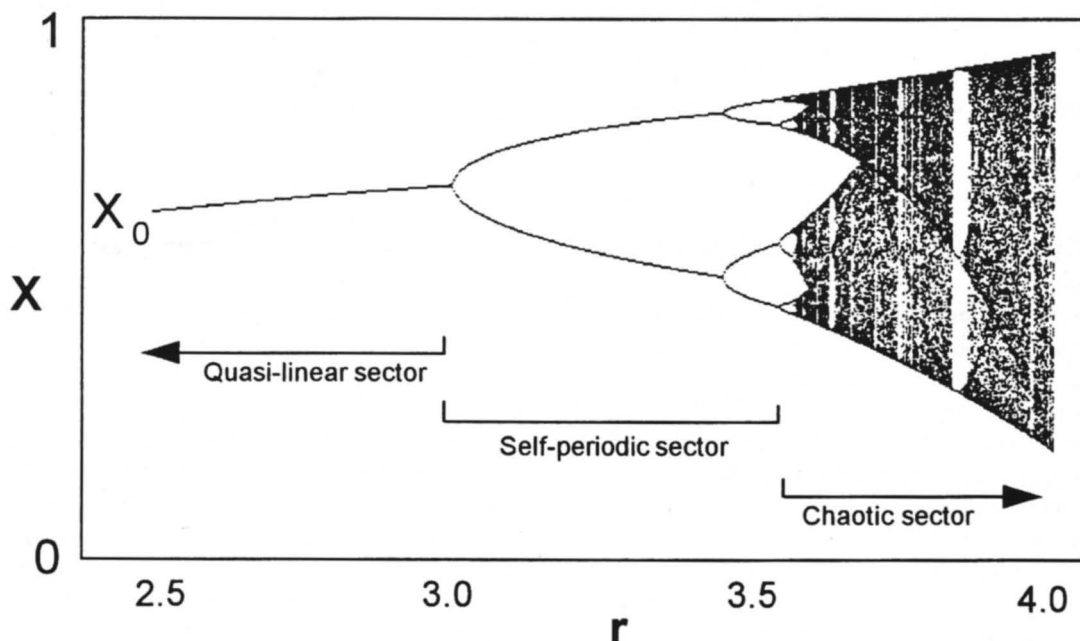


Fig. 1: The "Bifurcation diagram": one-dimensional map of the recursive iteration of the simplified logistic equation. The ordinate X represents the size of a certain population which varies between 0 (extinction) and 1. X_0 is the initial population size. The abscissa r simulates variable reproduction rates. The larger r is more the iteration system deviates from "linearity".

of the astronomical theory... but an extension of it" since it combines astronomical induced driving forces (*i.e.* Milankovitch theory) with aspects of nonlinear dynamics (*i.e.* Climate) particularly some recent developments of Chaos Theory.

The proxy nature of the micropalaeontological data we gather has special characteristics since we are dealing with living entities with complex ecological and physiological responses to their surrounding environment. Although calcareous nannoplankton are phytoplanktonic, present day studies have demonstrated that they are not usually light limited (Margalef, 1991). So, in our model the main Milankovitch influence will be centred on the increase/decrease of seasonality, related to stronger/weaker global temperature gradients which are directly (although nonlinearly) related to stronger climate induced turbulence (both oceanic - upwelling, and atmospheric - storm disturbance). Ultimately this turbulence controls nutrient input in the system. We will leave for the moment the question of exactly how the surrounding palaeoceanographic environment is physically reacting to orbital forcing².

As a paradigm of this model the Bifurcation Diagram of the logistic equation in its canonical (simplified) form will be used (May, 1974, 1976; May & Oster, 1976) (Eq. 1) as an example for the several situations that this model assumes as possible to occur in natural systems.

$$X_t = r \cdot X_{t-1} \cdot (1 - X_{t-1}) \quad (1)$$

Where X is the hypothetical population size at a certain time interval t , and r is a constant that simulates their reproduction rate per unit of time interval. However

it there is no assumption that natural systems are fully described by this or any other simplified equation, only that their behaviour, when iterated, may follow its basic features. In other words: the logistic equation is used here not so much as a potential model for population growth in an environment of limited resources but as a suitable (and common) example of how nonlinear systems work.

Even in this simplified version the logistic equation is a typical example of a nonlinear equation with a feedback mechanism, [the term $(1 - X_{t-1})$ in Eq. 1], that prevents populations from growing indefinitely. Fig. 1 is a one-dimensional map of the recursive iteration of this simplified logistic equation which is known as the "Bifurcation diagram". The recursive iteration of this equation is a mathematical procedure arguably analogous to a natural system in which the value of a given property at one time interval (X_t) is dependant on its value in the preceding time interval (X_{t-1}). The sequence of values produced can thus be analogous to any time series data set, for instance from sequential core samples, in which a certain sample is related to the previous ones by a sequence of several (natural) processes, repeating themselves on and on (Fig. 2).

Let's now assume that r (referred to above as the reproduction rate) modulates the increase in a population size, after a certain time interval, due to the availability in nutrients. At a first glimpse we could consider that when r increases the size of the population also increases, more or less, proportionally. Well, this is true but only for a short interval of r values.

Depending on the values of the parameter r a certain theoretical population (X) may converge to a

² - It is interesting to notice how recent models of the physical behaviour of certain palaeoceanographic features such as the North Atlantic conveyor (its thermohaline circulation) considers a "stochastic or random forcing" component (*e.g.* magnitude of freshwater flux forcing) (Weaver & Hughes, 1994) with no linear direct response to orbital forcing (obviously that there is a certain relationship since stronger seasonality due to Milankovitch cyclicity induces a stochastic hydrological cycle with larger standard deviation), mainly because changes in the conveyor circulation pattern have a much higher frequency and an unpredictable nature.

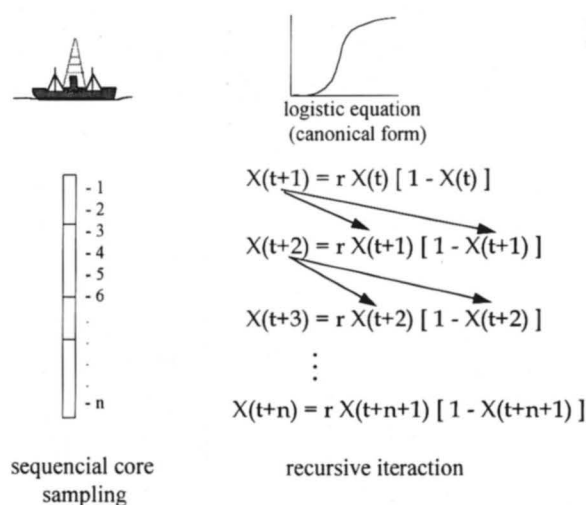


Fig. 2: Comparative diagram showing the similarity between the study of a sequential core sampling and the behaviour of a simple non-linear equation when we iterate it recursively.

unique size value (quasi-linear sector of the diagram, where X is approximately proportional to r), may self-oscillate between 2, 4, 8, 16 ... distinct size values (self-periodic sector, where "sudden" bifurcations arise after which the population size, X , "oscillates" between extreme lower and upper values) or may never converge (aperiodic or chaotic sector, where the size of the population loses predictability) (Fig. 1). By simply increasing r the behaviour of the population described by this simple equation diverges more and more from "linearity".

Possible palaeoecological - palaeoceanographic meanings of these three sectors are not yet fully understood but some interpretations are possible:

i) The quasi-linear sector represents the interval where the system reacts almost as a "transient" linear system, i.e. the outputs (X) are unique and almost proportional to the inputs (r). In this stage the system is quite robust since even after strong disturbances (a significative change in the initial size of the population) the system reacts in order to converge to the same final size value (Fig. 3). By other words strong "anomalous" environmental disturbances, by their magnitude, can induce direct unpredictable responses in the organisms (e.g. abundance drops, blooms, para-acmes, etc.) although in this stage the system responds to reach the initial stable value (something that a typical linear system cannot perform) and by doing that it better reflects the overall conditions. The model predicts that when the system is "working" in this way it should be possible to recognize in it external (orbital) signals and to apply Milankovitch models (Imbrie *et al.*, 1992). This behaviour is expected to occur for sites located in oceanic domain (more or less stable central ocean gyres) and for oligotrophic phytoplankton groups that have lesser needs for nutrients (e.g. Discoasters in sites far away from coastal areas, Backman, 1986; Backman *et al.*, 1987) since nutrient concentrations are more dependent on climate and turbulent transport;

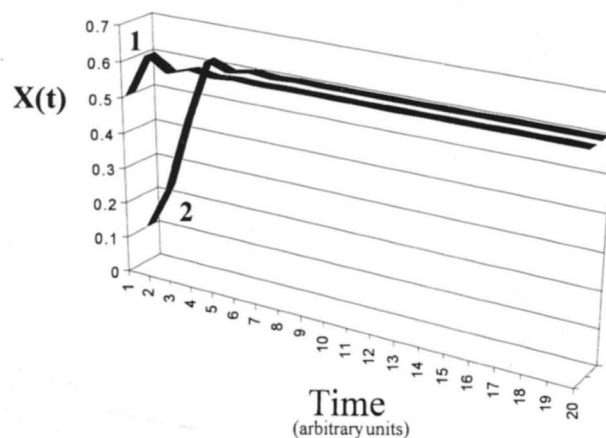


Fig. 3: Logistic generated time series: quasi-linear behaviour. 1 - initial state; 2 - behaviour after a strong disturbance (significant change in initial conditions).

ii) The self-periodic sector shows how a nonlinear system may disclose a rhythmic pattern without any need for external (cyclic or not) forcing. In this stage the system shows some sensitivity to disturbance as can be seen in Fig. 4. In some cases after a change in initial conditions the system may return to the same behaviour (Fig. 4A) while in other situations the system may respond with a small or a large phase shift (Figs. 4B and 4C, respectively). It is possible that some cases of rhythmic patterns in time series (e.g. limestone-marl or any other lithological couplet sequences) may result from this property of nonlinear systems. If so, instead of being a consequence of external orbital forcing they might correspond to an internal and also natural "heart beat". This could explain those cases when spectral analysis reveals non Milankovitch's frequencies or cases where we have a quasi-periodic behaviour (evidenced by a torus structure - doughnut shaped - in phase space) due to interference between distinct frequencies or disturbance of a spectral signal by defocus of a cyclic pattern (Fig. 4B and C) (see footnote);

iii) In the chaotic or aperiodic sector the system is intransient, never converges and shows an extreme sensitivity to initial conditions, the "Butterfly effect" of (Lorenz, 1964). In these conditions an undetectable change in the initial values (e.g. a shift of 0.02 arbitrary units) leads to an exponentially divergent situation after only a few iterations (Fig. 5). The system becomes unpredictable and several tests may be applied to determine the dimensionality of its attractors (in phase space) as a way to distinguish this chaotic system from a noisy time series. Computing Lyapunov exponents or running nonlinear predictability tests (Sugihara *et al.*, 1990; Sugihara & May, 1990) are some of the other methods that can be used to demonstrate the real nature of apparently random (chaotic) time series.

This last situation is expected to happen in time series retrieved from cores near continental areas where climate disturbances (run off) and upwelling are quite frequent. Several workers have already shown that both climate and turbulence have a typical chaotic behaviour and characteristic fractal properties (Mandelbrot, 1983; Margalef, 1991; Mullin, 1992; Palmer, 1992; Peixoto &

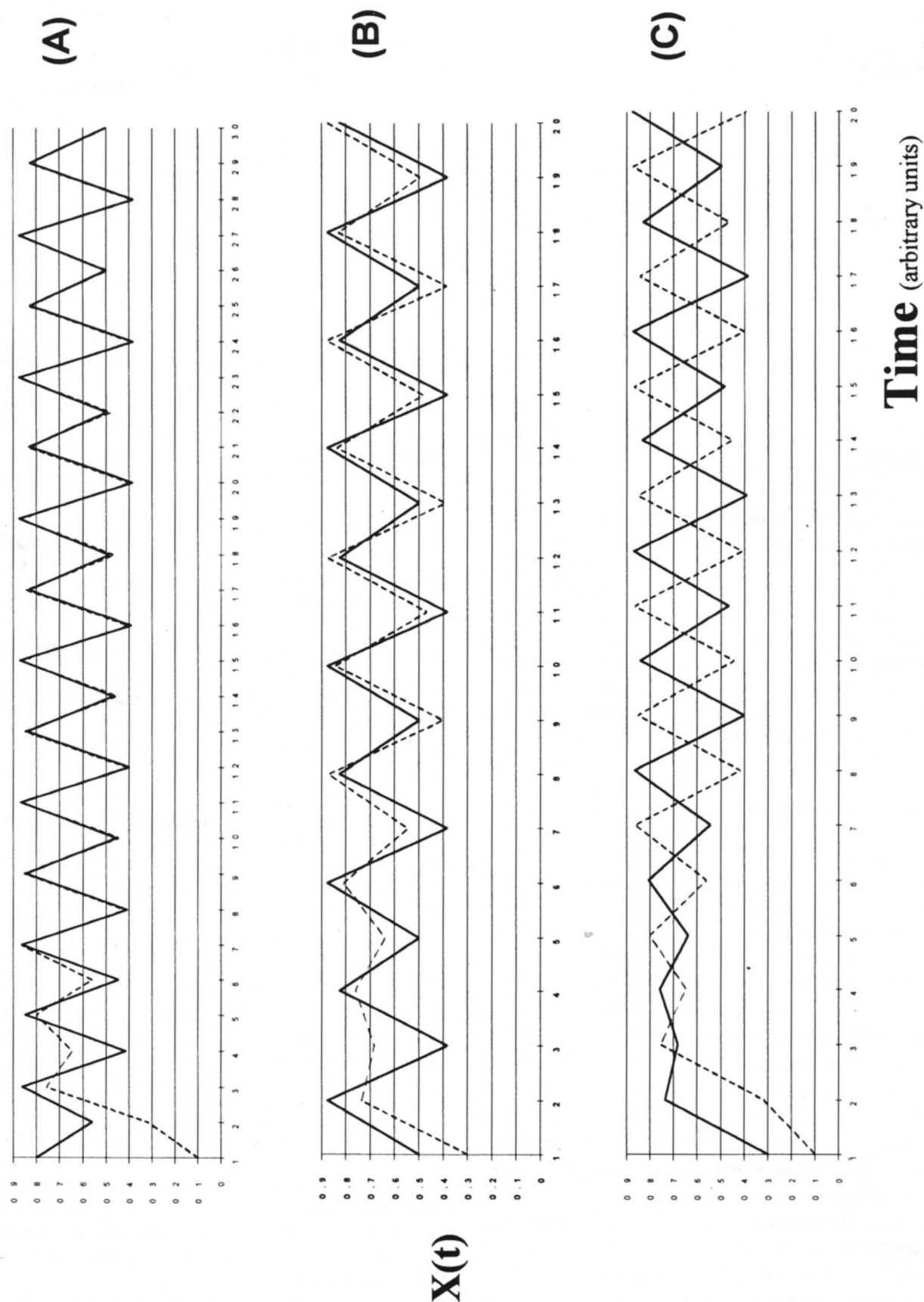


Fig. 4: Logistic generated time series: self-periodic behaviour. A - convergent behaviour; B - small shift of phase induced by disturbance; C - significant shift of phase induced by disturbance.

Oort, 1992), of the same basic nature as the chaotic sector of the Fig. 1. In fact the scale invariant (self-similarity) property of fractals can also be illustrated by this same

Bifurcation diagram in which some portions of the diagram (as for instance a small window in the vicinity of the limit between the self-periodic and chaotic sectors), when

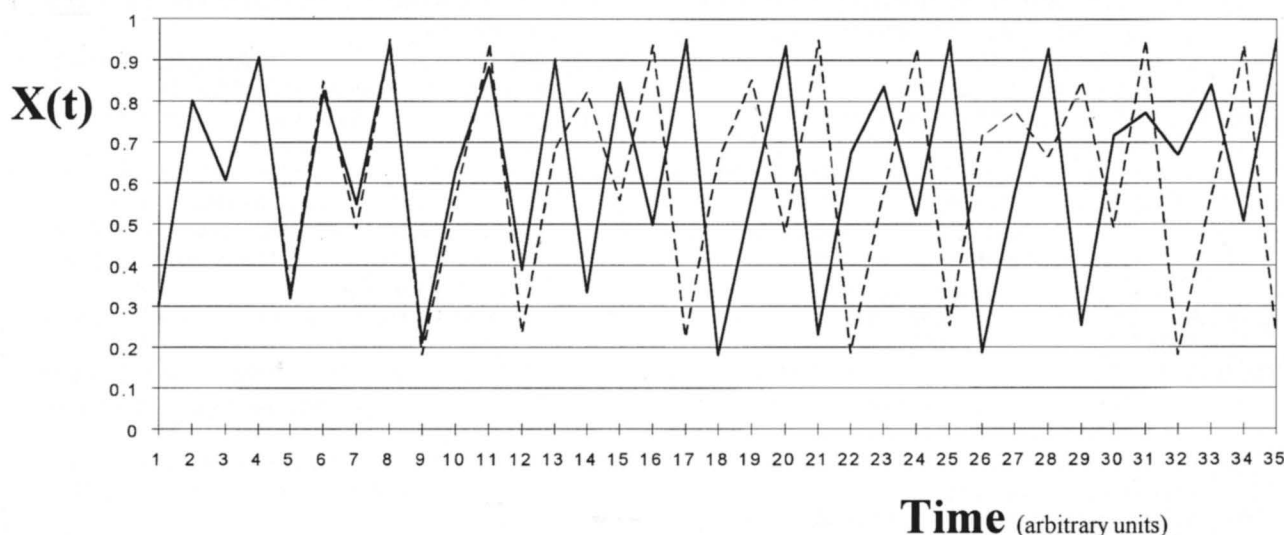


Fig. 5: Logistic generated time series: chaotic behaviour. After an "insignificant" disturbance of 0.02 (arbitrary units) the system changes completely after only few iterations. This is known by the "butterfly effect" or sensitivity to initial conditions which is one of the characteristics of the chaotic dynamics.

enlarged, reproduce the same basic features of the whole diagram (Fig. 6-A).

Also other properties of the Bifurcation diagram such as the "universality" (or the feigenvalue, the constant ratio between the r values which occurs at two contiguous bifurcations) or the "windows of organization" (the white strips in the chaotic sector, short moments at which the system starts to reconverge, Fig. 6-B) may also be interpreted (probably not so easily) in terms of how natural (palaeo)systems (population dynamics or turbulent motion) work (Stewart, 1990).

So far we have discussed only a single population but studies made on multi-dimensional nonlinear systems show that the chaotic sector increases and appears "sooner" than in the case shown in figure 1.

It has been demonstrated that "chaos does not rise because of noise in the system or imprecision of measurement. Instead it is an intrinsic feature of the physical [natural] system" (Crilly *et al.*, 1991).

The "Complex behaviour model" as presented above does not aim to "explain" exactly how and when the system shifts from the periodic orbital signal (in some cases, like in the "Younger Dryas", we may be able to understand why, but when we go further in the past our chance of pin-pointing every one of these possible "anomalous" cases is virtually impossible) but to try to understand, within a single framework (the nonlinear behaviour of the natural systems), several of the possible situations that might happen in the study of a time series proxy data. I believe Chaos theory and nonlinear dynamics applied to palaeoecology can help us to understand how even "simple" ecological systems may reveal a rather complex pattern, which quite easily could be interpreted as noisy and simply filtered and tuned to a predetermined Milankovitch frequency signal. We may miss much if we use this last procedure, blindly, as a routine.

I hope that this essay may lead to a general discussion (perhaps in a workshop on palaeoecology - palaeoceanography, in the next INA Conference) about the behaviour of natural systems, how they work and which are the best methods (for each possible case) to study them.

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³ - The Dryas was a short cold period during the last deglaciation, around 10,500 years ago, related to massive meltwater discharge of fresh water into the region of North Atlantic Deep Water formation, sufficiently large to stop conveyor circulation (Broecker *et al.*, 1988). This event is commonly referred as an example of an "high frequency oscillation in the geological record [which] cannot be explained by simple Milankovitch models" (Duplessy *et al.*, 1981).

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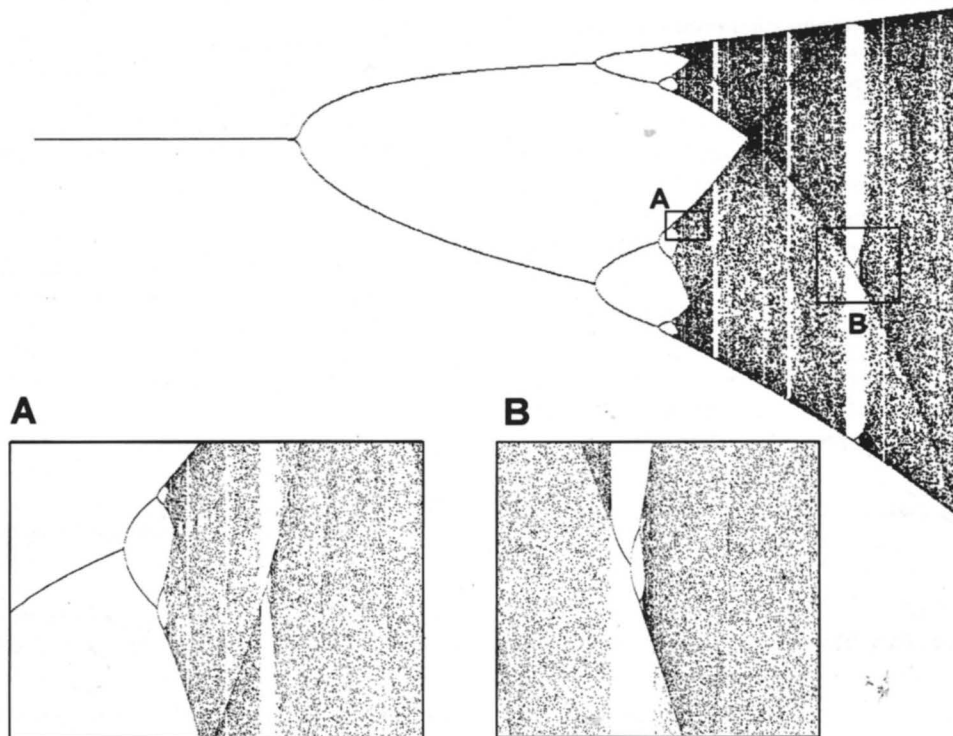


Fig. 6: Bifurcation diagram showing (A) its fractal self-similarity (when enlarged the small window reproduces the basic features of the entire diagram) and (B) its windows of organization (the system "suddenly" starts to converge again for a certain short interval of r values).